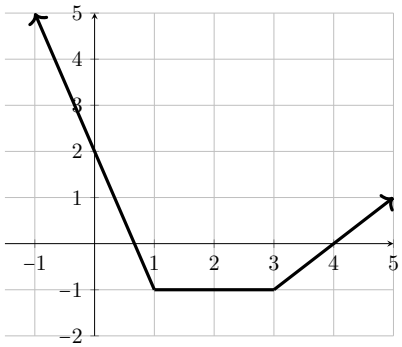
Turn in the following problems:

1. Use the function $f(x) = 4x^3e^x$ to answer the following problems:
   (a) On which interval(s) is the function $f(x)$ increasing?
   (b) On which interval(s) is the function $f(x)$ concave upward?

2. (a) If $F(x) = f(x)g(x)$, where f and g have derivatives of all orders, show that
   $$F'' = f''g + 2f'g' + fg''.$$
   (b) Find similar formulas for F''' and $F^{(4)}$.
   (c) Describe the pattern for higher derivatives of F.

3. A table of values for the functions $f(x)$ and $f'(x)$ and a graph of the piecewise linear function $g(x)$ are shown below.

x	$f(x)$	$f'(x)$
-1	11	-7
0	2	-2
1	-2	5
2	9	3
3	0	4
4	1	2



$g(x)$

   (a) Given $h(x) = f(x)g(x)$, find $h'(1)$.
   (b) Given $p(x) = \dfrac{f(x)}{g(x)}$, find $p'(2)$.
   (c) Given $q(x) = \dfrac{g(x)}{f(x)}$, find $q'(2)$.
   (d) Given $q(x) = \dfrac{f(x)}{g(x)}$, find $q'(3)$.
   (e) Given $\ell(x) = \dfrac{g(x)}{\sqrt{x}}$, find $\ell'(4)$.

4. Prove that $\frac{d}{dx}(\csc(x)) = -\csc(x)\cot(x)$.

5. Fill in the blank with "all", "no", or "some" to make the following statements true. Note that "some" means one or more instances, but not all.

   - If your answer is "all", then give a brief explanation as to why.
   - If your answer is "no", then give an example and a brief explanation as to why.
   - If your answer is "some", then give two specific examples that illustrate why your answer it not "all" or "no". Be sure to explain your two examples.

   An example must include either a graph or a specific function.

   (a) For _____ functions f and g, if $\frac{f(x)}{g(x)}$ is defined but not differentiable at $x = 1$, then either $f(x)$ or $g(x)$ is not differentiable at $x = 1$.

   (b) For _____ functions f and g, if f and g are two functions whose second derivatives are defined, then $(f \cdot g)'' = f \cdot g'' + f'' \cdot g$.

   (c) For _____ functions f and g, $(f(x) \cdot g(x))' = f'(x) \cdot g'(x)$.

   In mathematics, we consider a statement to be false if we can find any examples where the statement is not true. We refer to these examples as counterexamples. Note that a counterexample is an example for which the "if" part of the statement is true, but the "then" part of the statement is false.

6. A manufacturer produces bolts of a fabric with a fixed width. The quantity q of this fabric (measured in yards) that is sold is a function of the selling price p (in dollars per yard), so we can write $q = f(p)$. Then the total revenue earned with selling price p is $R(p) = pf(p)$.

   (a) What does it mean to say that $f(20) = 10,000$ and $f'(20) = -350$? Include units in your answer.

   (b) Assuming the values in part (a), find $R'(20)$ and interpret your answer.

These problems will not be collected, but you might need the solutions during the semester:

7. If f is a differentiable function, find an expression for the derivative fo the following function:

$$y = \frac{1 + xf(x)}{\sqrt{x}}$$

8. A ladder 10 ft long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \frac{\pi}{3}$?

9. Find the given derivative by finding the first few derivatives and observing the pattern that occurs.
   (a) $\frac{d^{99}}{dx^{99}}(\sin(x))$
   (b) $\frac{d^{35}}{dx^{35}}(x\sin(x))$

Optional Challenge Problems

How many tangent lines to the curve $y = x/(x+1)$ pass through the point $(1, 2)$? At which points do these tangent lines touch the curve?